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BUILDING AND PREPROCESSING OF IMAGE DATA USING INDICES OF REPRESENTATIVENESS AND CLASSIFICATION APPLIED TO GRANULAR PRODUCT CHARACTERIZATION

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SUMMARY

The characterization of granular products using image analysis is complex, as defining sample size is a very difficult task (should one use weight or number of particles?) and because of the diversity of the data which can be extracted from the image. A three-step procedure is applied: data extraction, data preprocessing and sample classification. We deal with the second step, once the image data have been extracted and gathered into histograms with a large number of intervals. The method we propose allows both the building of optimal size samples and the creation of data vectors appropriate for the third step. The originality of the method lies in the supervision of the data processing by taking into account the final goal, the discrimination into classes. Indices of stability and discrimination are created to build new histograms. To determine the optimal sample size, indices of representativeness and classification are used. This process has been tested on mill product images which are divided into three classes. The optimal sample size given by the representativeness index is 18 images, whereas it drops to 13 using the clasification index. For this example the features, if considered independently, are not informative enough to solve the problem (the best classification performance is 60%). It is necessary to develop a strategy where features are combined. This strategy is presented in a separate paper. © 1997 John Wiley & Sons, Ltd.

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KEY WORDS image analysis; granular product histograms; classification

INTRODUCTION

From the first milling of cereals to modern industrial activities such as cement making or pharmacology, humanity has been confronted with the problems involved in the fabrication of granular products. The two main difficulties are improving quality and increasing productivity. These two objectives are usually reached through automation. However, quality control has to take into account both objective and subjective criteria.[1] In many industries, quality control is carried out by an expert who controls all the mechanisms of production. He is guided not only by chemical and physical analysis but mainly by subjective characterization (vision, touch). To help this expert in this painstaking task and to improve the quality consistency of granular products, many researchers have proposed artificial methods[2,3] using different sensors. Image analysis seems to be the most promising

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one because it is fast and non-destructive and because there is no restriction on the type of products that can be analysed.

Two kinds of features can be extracted from images of granular products: individual features (such as surface area, perimeter length, etc.) and global features relative to image texture. The first set is extracted from isolated particles and leads to a statistical distribution of parameters called ‘histogram features’. The second set corresponds to measurements on the whole image. These two types of features are fundamentally different are described by vectors of different sizes. These sizes are the same for all the ‘individual features’ and heterogeneous for the ‘global features’. Most researchers[4] limit themselves to a single type of extracted feature to solve their classification problem. However, in the framework of a classification problem as complex as a qualitative evaluation the information contained in either type of feature may not be sufficient by itself.

The objective of this research is to provide a general approach to classify populations of granular products by using different types of features extracted from the image.

The first step of this process deals with evaluating the possible features to be extracted from images of granular products. Then the classification of granular population can be considered as a standard pattern recognition problem with some specific difficulties. The major one is related to the number of elementary particles required to develop and test the classification system, i.e. the number of images that must be aggregated to define a sample. This number should be sufficiently large to be representative of the population, but without being completely representative this number can be smaller once the classes can be distinguished from one another. In order to reach real-time processing speed, the sample size to distinguish between classes has to be minimized.

Most of the features are histograms. Generally the histogram intervals are chosen *a priori*. This can be dangerous in the framework of classification, because if the number of histogram intervals is too large, the frequency values may not be statistically valid, and if too small, some pertinent information may be eliminated. Therefore a strategy for automatically generating the histogram intervals according to the classification performance was developed.

The large size of the feature vectors and the noise contained in them may be an obstacle to developing a robust system. Therefore it seems necessary to reduce their dimensions as much as possible but without losing relevant information concerning the classification problem.

METHODS

Definitions

Our research is drawn from the domains of image and statistical analysis. It is applied to flour and products from other milling industries. At the entrance of a classical grain mill, two sets of rolls, B1 and B2, are successively used to grind the wheat. In the experiment we create three quality classes by adjusting the roll gap of B2.

The following definitions are given as an introduction.

For each of the three quality classes, 350 images are recorded. A sample is made by aggregating N images of the same class. N must be defined at the beginning of the classification procedure according to the product (wheat, maize, etc.). Once defined, it is constant for all the classes. Therefore in each class the training set contains $350/N$ samples.

Features such as object area or elongation are image attributes; 21 features were identified (see Table 1 in ‘Experimental’ section). For each class a matrix of feature histograms was built containing one column per sample (i.e. $350/N$ columns) and one row per feature (i.e. 21 rows).

Features to be extracted

The features are extracted as described by Guillaume *et al.*[5] and used for preprocessing in order to reduce their size and to standardize them. This preprocessing and the number of images to define a sample (i.e. N) are optimized according to several figures of merit, i.e. 'stability' and 'discrimination' indices for histograms and 'representativeness' and 'classification' indices for the number of images. The new histograms should be shorter (fewer intervals), as representative and more discriminating than the raw ones. The numbers of images must be as small as possible according to real-time processing needs, but large enough to be representative.

General procedure of data preprocessing

The procedure is iterative and combines the two objectives.

1. Given the choice of two features (in the present example, 'object area' and 'length along largest-variance axis/object perimeter'), the following steps will be applied to these two model features.
2. Initialization of the number of images, N.
3. Building of cumulated histograms of the two model features for N images.
4. Reduction of the dimension of these histograms according to 'stability' and 'discrimination' indices.
5. Test of the 'representativeness' and 'classification' capacities of the reduced histograms.
6. If the test is faulty, N is incremented and the procedure is started again at step 3.
7. If the test is valid, i.e. $N_{\text{optimal}}=N$ (number of images), the cumulated histograms of all the features are built on N_{optimal} images.
8. Reduction of the dimension of all the histograms according to stability and discrimination indices.

Because the figures of merit deal with measurements of distances between histograms, it is necessary to define a metric.

The metric

Chi-squared (χ^2)[6] seems to be an appropriate metric when using histograms. The principle is to determine whether a histogram can fit a theoretical distribution model. In our case the theoretical distribution model can be built as the average of the two histograms to be compared.

Let us consider that n and n' are the numbers of elementary particles represented by the two histograms H_i and H_i' respectively and T_i is the number of particles in interval i. This is to be applied to the two histograms and to the theoretical distribution model respectively.

The chi-squared value is defined as

$$\chi^2=\sum_{i=1}^{s}\frac{(T_i-H_i)^2}{T_i}$$

with s being the number of intervals in the histogram.

Let us define

$$T_i=\frac{H_i+H_i'}{2}$$

The chi-squared value becomes

$$\chi^2=\frac{1}{2}\sum_i\frac{(H_i-H_i')^2}{H_i+H_i'}$$

The frequency associated with interval i of histogram H is $f_i=H_i/n$.

Assuming that the numbers of particles in the two histograms are close to one another ($n\approx n'$), the chi-squared value can be given according to the elementary frequencies:

$$\chi^2=\frac{n}{2}\sum_{i=1}^{s}\frac{(f_i'-f_i)^2}{f_i'+f_i}$$

In order to be independent of the population size, the sum should be the only element considered:

$$\Delta=\sum_{i=1}^{s}\frac{(f_i'-f_i)^2}{f_i'+f_i}$$

In the following, Δ is used to measure the distance between two histograms of s intervals.

Automatic building of distribution intervals for histograms

The initial histograms are made using 100 intervals. The intervals are selected to give a linearization of the cumulated histogram to build equally populated intervals. The number of intervals is then decreased step-by-step by concatenating two adjacent intervals at each step. To select the most appropriate intervals, two indices are proposed: one to characterize the stability of the reduced histogram, the other to predict its discrimination capacity considering the quality classes of the particles. Both indices are computed using a clustering method.

Clustering method

The clustering method is based on the mobile centre algorithm.[7] For each cluster a centre of gravity, i.e. a sample from the training set, is randomly chosen. Each remaining sample of the training set is assigned to the nearest cluster. In order to do this, we compute the distance between this sample and the various centres of gravity using the metric defined above (chi-squared).

When all the samples are classified the final centre of gravity of each cluster is computed. The error is the sum, for all the clusters, of the distances between the initial (random) and final centres of gravity.

This procedure is repeated until the error drops to a threshold value. This threshold value depends on the metric used for the distance calculation, on the number of clusters in use and on the number of elements in the training set.

The number of clusters is defined as the square root of the number of elements in the training set. Having more clusters than classes ensures that the procedure is not dependent on the class shapes and increases the resolving capacity of the system. For an initial training set size of 100 samples the number of clusters was ten and the threshold value was 0·001.

Stability index

To measure the stability, an index is computed within each class. This index quantifies the change in the histogram when two intervals are concatenated.

Let us consider a histogram made of K intervals. The histogram reduction (from K to $K-1$ intervals) consists of concatenating two adjacent intervals. All the possible concatenations are achieved. Then the clustering method is applied to the initial histogram (size K) and to each reduced histogram (size $K-1$). For each case a similarity matrix is computed. It is a square matrix, the dimension of which is equal to the number of elements in the training set. The i, j-cell is set to '1' if the samples are assigned to the same cluster or set to '0' if not.

Because the clustering method is not accurate and can lead to very different partitions (depending on the initial gravity centres), it is applied several times using different random initializations (ten in the present example). The final similarity matrix is the sum of the ten individual similarity matrices. Thus the cell maximal value is the number of clustering repetitions (ten in our case).

For a given concatenation S_k the stability index is the 'error value' between its similarity matrix and the similarity matrix of the initial histogram. S_k is the sum of squares of the difference between the values of the same cells in the two matrices:

$$S_k = \sum_{i,j} (I_{[i][j]} - I_{k[i][j]})$$

where i and j vary from 1 to N, the number of elements in the training set, I is the similarity matrix associated with the initial histogram (size K) and I_k is the similarity matrix associated with the concatenation of intervals k and $k+1$ (k ranges from 1 to $K-1$).

The selected configuration is the one that gives the minimum stability index. If there are several equal minima, the one which minimizes the variance of each interval is chosen.

The similarity matrix I_k associated with the selected configuration becomes the similarity matrix I for the next step.

Discrimination index

This index deals with the use of histograms to discriminate between classes based on quality, which are the outputs of the classifier. The training set covers all the classes. The discrimination index aims to measure the homogeneity of the clusters.

As for the previous index, consider a histogram of size K. The number of elements in the training set, the number of clusters in use and the number of classes are given.

Clusters are built for each of the $K-1$ possibilities of merging. For each cluster a normalized frequency array is computed. Its size corresponds to the number of quality classes. For example, if a given cluster contains ten samples, if the number of possible quality classes is four and if two of its elements belong to class 1, five of them to class 3 and three of them to class 4, the normalized frequencies are $f(1)=0{\cdot}2$, $f(2)=0$, $f(3)=0{\cdot}5$ and $f(4)=0{\cdot}3$.

The heterogeneousness criterion associated with a cluster c is

$$J_c = \sum_{i=1}^{q} \sum_{j=i+1}^{q} f(i) * f(j)$$

where c is the cluster number, q is the number of classes and f_i is the occurrence frequency of class i in cluster c. For instance, the heterogeneousness criterion is 0·31 for the example given above. This criterion is equal to zero for clusters made up of elements from the same class.

The discrimination index D is the weighted sum, for all the clusters, of the heterogeneity criteria J, the cluster weight value being equal to the number of samples contained in each cluster:

$$D = \frac{\sum_{c=1}^{C} W_c * J_c}{\sum_{c=1}^{C} W_c}$$

where C is the number of clusters in use, W_c is the weight associated with cluster c and the denominator is the number of elements in the training set, N. The lower the index, the better is the discrimination. As for the stability indicator and for the same reasons, the index is the result of the repetition of several experiments using different random initializations.

Combination of the two indicators

At the beginning of the procedure of histogram reduction it is useful to consider the stability index alone, because the intervals are not always statistically representative. Then the two indices can be combined and the reduction can be achieved using only the discrimination index.

There are are least two ways of combining indices. The first one is a weighted sum of the two indices; the selected configuration would be the one corresponding to the minimum sum. Another way could be to sort the configurations in decreasing order following the two indices and finally to select the first two common ones.

Determination of sample size

The number of images required to make a valid sample can be computed according to a representativeness criterion but also according to a classification criterion.

Representativeness criterion

We can consider that a sample made of N images is representative of a population when the cumulated histogram of these N images is stable. Thus, if two samples are constructed from images randomly chosen in the same class, the two histograms must be similar.

The proposed procedure is iterative. The number of images taken into account to build a sample is increased as long as the distance between the histograms of two samples of the same class is higher than a threshold value. For a given number of images the process is carried out a given number of times. This is defined as the 'number of experiments'. The representativeness criterion is the percentage of success, i.e. the times when the distance between the two histograms is lower than the distance threshold. The procedure ends when the representativeness criterion is higher than the success threshold. The parameters of the procedure are the distance threshold, the number of experiments and the success threshold (in %).

Classification index

The first approach consideres only two samples taken from two different classes ('two-sample' approach). Each sample is made up of X images. If X is not large enough, the histograms can be considered the same (considering the chi-squared distance) even though they belong to two different classes. The number of images is increased until this hypothesis can be rejected, i.e. until the histograms cannot be considered the same.

The number of images may also be defined using the '*k*-nearest-neighbour' method.[8] For a given sample the k nearest neighbours are determined using the metric previously described. If the classes are adequately separated from one another, the k neighbours of a sample should belong, to a large extent, to the same class: the percentage of neighbours belonging to the same class as the sample is called the 'neighbour percentage'. The number of images to build a sample is increased until the proportion of neighbours of a same class reaches a threshold value. At each step this procedure is repeated several times (defined as the 'number of experiments'). For each experiment the samples are rebuilt by drawing lots, which makes this approach time-consuming. The 'neighbour performance' criterion is the percentage of success, i.e. the times when the neighbour percentage is higher than the threshold. The procedure ends when the neighbour performance criterion is higher than the success threshold.

The last step before using the histograms as input for the classifier is factor analysis, either PCA (principal component analysis) or FCA (factorial correspondence analysis)[9] according to the nature of the features. The new components are built by linear combinations of the histogram frequencies to maximize the remaining variance. Retaining only the first factors, the information is kept whereas the noise is removed.

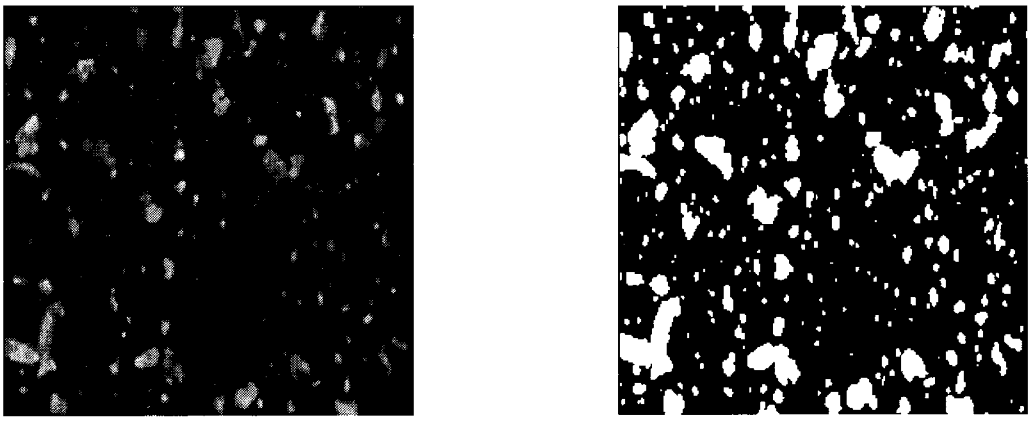
EXPERIMENTAL

Procedures

By adjusting the roll gap to 300, 400 and 500 μm, three quality classes of mill products are created. These classes are denoted E_p, where p is the roll gap (E*30*, E*40*, E*50*). For each class, 350 images have been stored, each of them containing about 120 elementary particles.

Image grabbing and analysis

As already described by Guillaume *et al.*,[5] (256 × 256)-pixel, 256-grey-level images are processed by custom software written using the C++ language in order to improve them (noise reduction, etc.) and



(a) (b)

Figure 1. (a) Grey-level and (b) binary images of granular product

Table 1. Features extracted from images and their size (for more details see Reference 5)

Var	Feature	Size	Var	Feature	Size
1	Object area	100	2	Object perimeter	100
3	Convex hull area	100	4	Convex hull perimeter	100
5	Var1/Var3	100	6	Var2/Var4	100
7	$Var2^2$/Var1	100	8	$Var4^2$/Var2	100
9	Elongation	100	10	Centred luminance moment order 1	100
11	Centred luminance moment order 2	100	12	Centred luminance moment order 3	100
13	Length along largest-variance axis/$\sqrt{Var1}$	100	14	Width long second-variance axis /$\sqrt{Var1}$	100
15	Length along largest-variance axis/Var2	100	16	Width along second-variance axis /Var2	100
17	Run length parameters	15	18	Run length histogram features	132
19	Vo-occurrence parameters	15	20	Co-occurrence histogram features	72
21	Opening grey-level distribution	9			

to extract the individual and global features (Figure 1). These features are listed in Table 1: Var1–16 correspond to individual features; Var17–21 correspond to global features.

RESULTS

Sample size research

The number of images used to build each sample was initially set at four. Histogram intervals were generated for two features. As experts are particularly concerned about particle size and visual information, we chose variables relating to size and shape. Thus 'object area' (Var 1) and 'length along largest-variance axis/object perimeter' (Var15) were chosen.

The threshold values which determine when the procedure should end in order to assess the sample size are not severe, because the information provided by each feature is limited. Combining information from all the features is likely to lead to better results.

Concerning the representativeness index, the three parameters are

(a) distance threshold, 0·8
(b) number of experiments, 40
(c) percentage of correct score, 75%.

The numbers of required images are given in Table 2.

The classification index was computed using the two approaches (i.e. 'two-sample' and '*k*-nearest-neighbour' methods) with the following parameters.

Table 2. Number of images required to make a sample according to representativeness index

Class	Var1	Var15
E30	13·45	17·5
E40	10·54	14
E50	10·9	6

Table 3. Number of images required to make a sample according to classification index (two-sample approach)

	Var2		Var15	
Class	Test 1	Test 2	Test 1	Test 2
E30–E40	8·5	7·5	8·5	5·5
E40–E50	18	16	18·5	14
E30–E50	5	4	5	5

Two-sample method

(a) Test 1. Threshold error set at 10% (severe parameter).
(b) Test 2. Threshold error set at 20% (less severe).

Because it is less severe, it is obvious that Test 2 will lead to a lower number of images than Test 1 (see Table 3).

K-nearest-neighbour method

In the present example the parameters defined in the ‘Classification index’ section are set to the following values:

(a) number of nearest neighbours, $x=8$
(b) neighbour percentage threshold, 70%
(c) neighbour performance threshold, 80%.

These results (Table 4) show that the representativeness and classification indices do not lead to similar conclusions. For the representativeness index the required number of images is twelve on average for the two considered features, while it is about eight or 8·5 depending on the classification index used (two-sample or *k*-nearest-neighbour method).

These averaging procedures hide some important aspects. E30 is the most difficult class to stabilize but the easiest to separate from the others. However, a large image number is necessary to discriminate classes E40 and E50, although they can be easily stabilized. In fact, the difficulty would appear to come from class E40, while class E50 seems to be homogeneous.

The number of required images may be determined not by the average of each table but rather by the maximum value. This number is 17 according to the representativeness index (Table 2) and ranges from 13 (Table 4, *k*-nearest-neighbour approach) to 18 (Table 3, two-sample approach) according to

Table 4. Number of images required to make a sample using classification index (*k*-nearest-neighbour approach)

Class	Var1	Var15
E30	10·3	7·5
E40	13·6	11·5
E50	6·1	5

the classification index. The number 18 is given for classes E40 and E50 which are very close; for the other pairs it drops to eight using the same index. This is why we propose to choose 13 images to make a sample.

Histogram interval generation

For the individual features (Var1–16) the number of histogram intervals has been initialized at 100. In the iteration process to reduce the number of intervals, the stability and discrimination indices are used successively. In fact, stability is the most important parameter, because if the classes are not stable, they cannot be used for discrimination. The discrimination index can be used further, but as it is time-consuming, it is preferable not to use it too early. Preliminary experiments on the selected features enabled us to find a good compromise between performance and time consumption.

(a) From iteration 1 to 60 the stability index is used exclusively.
(b) From iteration 60 to 70 the two indices are combined.
(c) For the remaining iterations the discrimination index is applied exclusively.

At this stage the number of remaining intervals is the one which conforms to a local minimum of the discrimination index.

Once this optimal procedure has been defined, it is applied to all the features. Table 5 illustrates how the number of histogram intervals have been reduced, for each feature, by this process.

Table 5. Reduction of size of features, i.e. number of histogram intervals ($n0$, original size; $n1$, size after automatic generation of histograms; $n2$, size after factorial analysis)

Var	$n0$	$n1$	$n2$
1	100	9	5
2	100	6	3
3	100	5	2
4	100	10	5
5	100	7	4
6	100	7	4
7	100	7	4
8	100	5	3
9	100	6	4
10	100	7	4
11	100	6	3
12	100	7	4
13	100	6	3
14	100	10	2
15	100	7	4
16	100	6	3
17	15	15	4
18	137	14	2
19	15	15	4
20	67	18	2
21	10	10	3

Use of factorial design

PCA is applied to individual features, Var1–16, and FCA is applied to global features (Table 5).

The observation of the training patterns projected on the factorial design, i.e. the principal component axes, gives us some clues as to the information contained in the individual features. Figures 2 and 3 show the projection of the training examples on the factorial design, where the variance percentage of each axis is noted. The two major axes do not account for all the variance and are not sufficiently wide-ranging to discriminate between classes (especially if the total variance accounted for is low). On the contrary, if the variance represented in the factorial design is large, the discrimination may be good. This is the case for Var17 in Figure 4, where the three classes can be quite easily discriminated. Table 5 also demonstrates that FCA helps to reduce the number of intervals.

Individual performances

In order to test the relevance of each feature on its own, the *k*-nearest-neighbour method was used.

The results obtained on the training base (with $k=8$) are summarized in Table 6. We have distinguished three types of decisions: correct, incorrect and ambiguous. We have an ambiguous output when a piece of data activates the outputs of two classes. Some features are relevant, such as Var19 or Var20, but their performances in classification are not satisfactory enough (64% of correct classification using the training samples). It should be noted that the features related to texture are more relevant than those related to size or shape. This is because texture parameters take into account both shape and size features. This result can also be linked to the expert global evaluation.

Table 6. Individual results of the features using the *k*-nearest-neighbour method

%	*Correct*	*Bad*	*Ambiguous*
var1	30.9	23.8	45.2
var2	42.8	23.8	33.33
var3	64.2	14.2	26.1
var4	52.3	11.9	35.7
var5	30.9	47.6	21.5
var6	30.9	47.6	21.5
var7	50	21.4	26.1
var8	33.3	50	16.6
var9	33.3	50	16.6
var10	28.5	47.6	16.6
var11	33.3	50	16.6
var12	47.6	26.1	26.1
var13	52.3	11.9	38.09
var14	45.2	19.04	23.8
var15	47.6	26.1	28.5
var16	45.2	16.66	38.09
var17	57.1	19.04	23.8
var18	52.3	16.66	33.33
var19	64.23	14.2	23.8
var20	64.23	14.2	23.8
var21	57.1	19.04	23.8

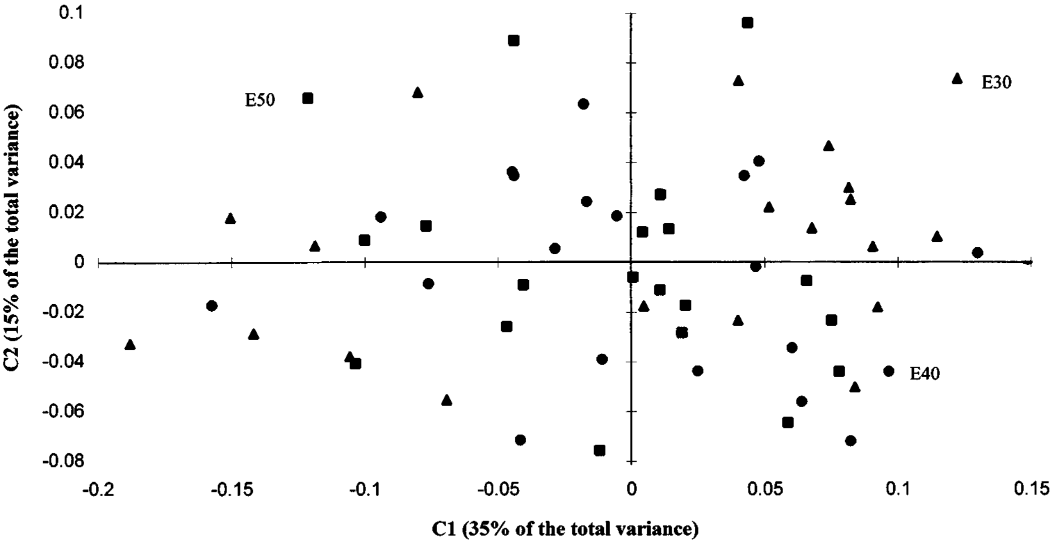


Figure 2. Projection of samples on factorial design (for Var6)

CONCLUSIONS

The image processing which is proposed for granular product classification consists of three steps: feature extraction, preprocessing and classification. This paper is principally devoted to the second

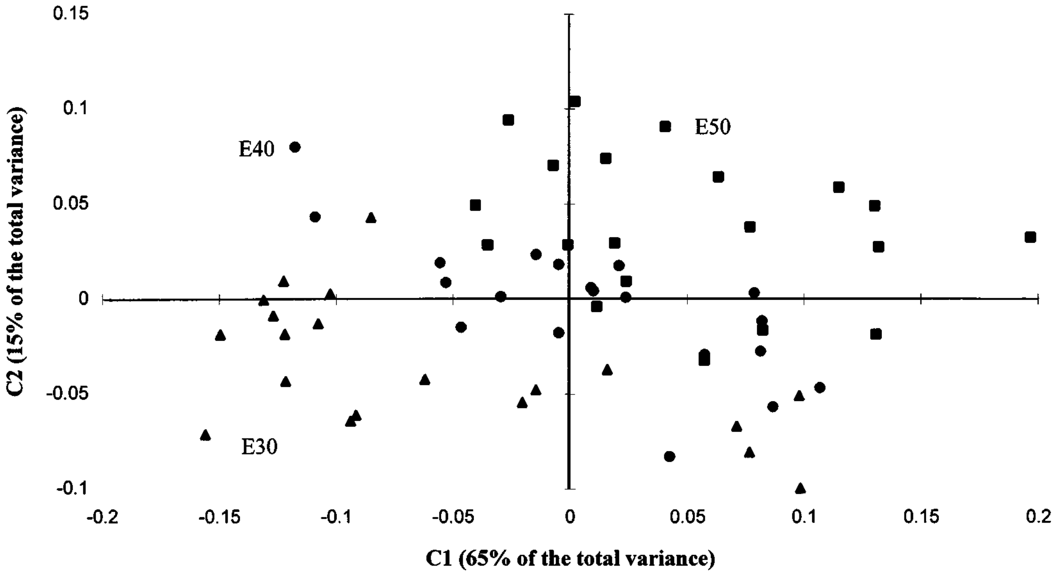


Figure 3. Projection of samples on factorial design (for Var15)

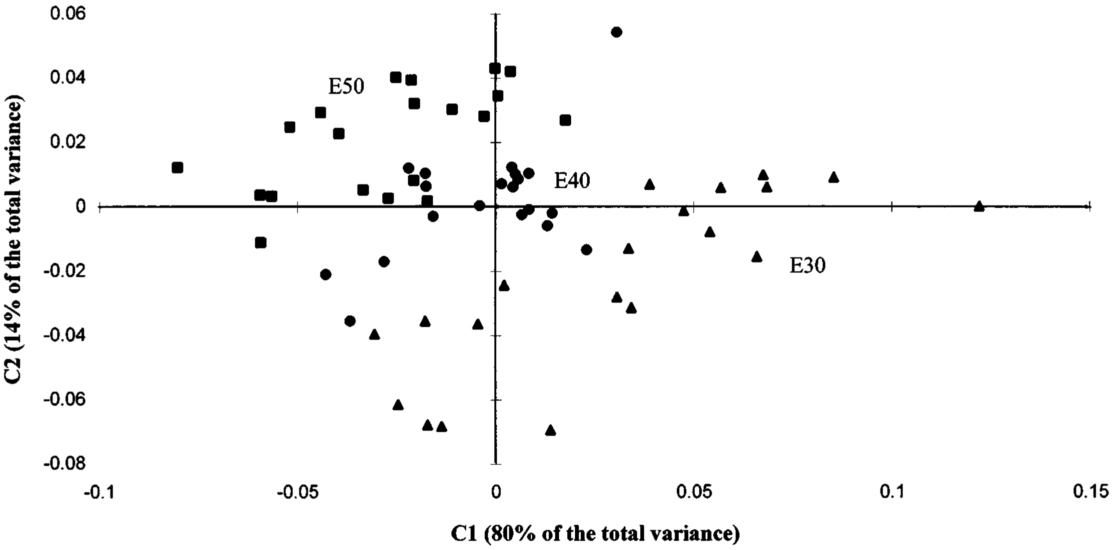


Figure 4. Projection of samples on factorial design (for Var17)

phase: the objective of the present work is to propose an automatic method for preprocessing images of granular products with the aim of further classification.

The first step is to set up a method to optimize the number of images that gives rise to a representative and discriminating sample.

New developments are subsequently described on how to design a more efficient classification system. The proposed iterative method to process the histogram classes is crucial. Histograms as feature vectors provide more information, but they can lead to poor classification when the number and width of the intervals of the histograms are chosen *a priori*. In the approach described above, the optimization of these intervals is carried out and the particularities of each classification problem are taken into account.

When resolving an existing mill classification problem, these techniques have led to an accurate classification of approximately 60% when the classification is achieved with one feature at a time. This rather poor performance is because each feature is not informative enough. The next step in this general resolution approach is to develop a method to combine the different features taking into account their heterogeneousness, their redundancy and their non-equal relevance for the classification problem. Accurate classification performances of up to 80% have been obtained using these new procedures. This work is presented in a separate paper.

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